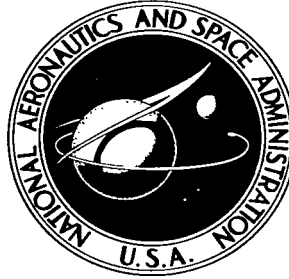


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**EXPERIMENTAL INVESTIGATION OF A  
3.0-INCH-MEAN-DIAMETER FULL-ADMISSION  
THREE-STAGE TURBINE SUITABLE  
FOR SMALL AUXILIARY POWER UNITS**

*by Robert Y. Wong and William J. Nusbaum*

*Lewis Research Center  
Cleveland, Ohio*



EXPERIMENTAL INVESTIGATION OF A 3.0-INCH-MEAN-DIAMETER  
FULL-ADMISSION THREE-STAGE TURBINE SUITABLE FOR  
SMALL AUXILIARY POWER UNITS

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NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

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SUMMARY

An experimental investigation was made of a 3.0-inch-mean-diameter full-admission three-stage turbine designed for an equivalent specific work output of 43.6 Btu per pound, an equivalent weight flow of 0.060 pound per second, and an equivalent rotative speed of 30 000 revolutions per minute. The performance of this turbine is compared with that of a reference two-stage turbine designed for the same requirements and with 31.34 percent admission in the first stage and full admission in the second stage.

A total-to-static efficiency of about 0.64 was obtained for the three-stage turbine at design blade-jet speed ratio. This value of efficiency compares favorably with a value of 0.65 calculated from an aerodynamic efficiency of 0.69 to account for large leakages through the second- and third-stage stator seals.

An experimental investigation of the first stage as a single-stage turbine, was conducted to verify the level of loss coefficient for turbines in this size class. At the design blade-jet speed ratio of 0.357, the total-to-static efficiency was 0.59 compared with 0.60 used in the design. This close agreement between the experimental and design efficiencies of the first stage verifies the level of loss coefficient used in the design of the turbine. These experimental results further indicate that, if the seal leakage is reduced, the three-stage turbine should perform near the design aerodynamic efficiency of 0.69.

Comparison of the performance of the three-stage turbine with the reference two-stage turbine indicated that the three-stage turbine with the high leakage seals performed as well as the best of the various configurations of the two-stage turbine. Further, it is expected that the three-stage turbine, with seal leakages reduced to reasonable levels, would be three to four points higher in efficiency than the two-stage turbine.

## INTRODUCTION

In many turbine applications for high-specific work output and low weight flow, it is necessary for the designer to compromise many factors in order to obtain an optimum design. If the designer selects partial admission, he can utilize larger diameters than if a full-admission design were selected. The advantage of using partial admission is that reasonable blade heights (greater than 0.1 in.) may be obtained with fewer stages. The use of a partial-admission first stage and full-admission second stage, as in reference 1, has been shown to have losses due to the large circumferential gradients into the second stage. These losses decreased with an increase in axial distance between stages. The reduction in loss was attributed to improved flow conditions at the entrance to the second stage. These circumferential gradients can also cause high radial loads on the bearings and possible turbine case distortions.

The full-admission turbine, on the other hand, requires a smaller diameter in order to have reasonable blade heights. The reduced diameter results in a turbine with a greater number of stages and possibly greater losses due to low aspect ratio and tip clearance effects. Efficiency predictions, such as those in reference 2, indicate that the performance of a full-admission turbine designed for the same work output and rotative speed as the turbine of reference 1 will be somewhat better than the partial-admission design.

The objective of this investigation was to design and to investigate experimentally a full-admission turbine designed for an equivalent specific work output of 43.6 Btu per pound, an equivalent weight flow of 0.060 pound per second, and an equivalent rotative speed of 30 000 revolutions per minute, which are the same requirements as those of reference 1. The turbine has three stages, and its mean diameter is 3.0 inches. The design and experimental investigation of the subject three-stage turbine and of the first stage as a single-stage turbine are presented herein together with a comparison with the 3.75-inch-mean-diameter two-stage partial-admission turbine of reference 1.

## TURBINE AERODYNAMIC DESIGN

### Design Requirements

The design work, rotative speed, and weight flow of the subject 3.0-inch-mean-diameter three-stage turbine are the same as those for the two-stage partial-admission turbine of reference 1. The aerodynamic design requirements are as follows:

Overall equivalent specific work output, $\Delta h/\theta_{cr}$ , Btu/lb . . . . .	43.6
Overall blade-jet speed ratio, $\nu$ . . . . .	0.221
Equivalent weight flow, $(\epsilon w \sqrt{\theta_{cr}})/\delta$ , lb/sec . . . . .	0.060
Overall speed-work parameter, $\lambda$ . . . . .	0.14
Equivalent horsepower, $\epsilon hp/\delta \sqrt{\theta_{cr}}$ . . . . .	3.7
Equivalent mean-blade-section velocity, $U_m/\sqrt{\theta_{cr}}$ , ft/sec . . . . .	392.8

All symbols are defined in appendix A.

## Velocity Diagrams

The free-stream velocity diagrams, computed to meet the design requirements, are based on the following assumptions:

- (1) Equal values of speed-work parameter per stage of 0.42
- (2) A one-third and two-third split of total-pressure loss between stator and rotor, respectively, for each stage
- (3) Equal rotor-inlet and -outlet relative critical-velocity ratio
- (4) Interstage seal leakage less than 1 percent of total flow with its effect on velocity diagrams negligible

Design total-to-total efficiencies of 0.63, 0.67, and 0.69 were used for the first, second, and third stages, respectively, which result in an overall total-to-total efficiency of 0.71. The total-to-total efficiency for each stage was obtained from an experimentally obtained loss coefficient and equation (8) of reference 2. The loss coefficient  $C_b$  was obtained from the experimental data presented in reference 3. In computing the aerodynamic efficiency for the subject turbine, the loss coefficient was adjusted for Reynolds number and nozzle-exit flow angle. The corresponding design total-to-static efficiencies are 0.60, 0.63, and 0.62 for the first, second, and third stages, respectively, with a value of 0.69 for the overall total-to-static efficiency. The resulting design overall total-to-total pressure ratio and overall total-to-static pressure ratio are 10.60 and 11.78, respectively.

The free-stream velocity diagrams, with a sketch of the blading showing the station nomenclature, are shown in figure 1. Flow angles at the stator exit as measured from the tangential direction are  $11.55^\circ$ ,  $14.20^\circ$ , and  $16.90^\circ$  for the first, second, and third stages, respectively. Stator-exit critical velocity ratios for the first and second stages are 0.867 and 0.938, respectively. The air flow at the exit of the third stage stator is slightly supersonic at a critical-velocity ratio is 1.032. The three rotors are designed for equal rotor inlet and exit mean-radius critical-velocity ratios  $W/W_{cr}$  of 0.518, 0.578, and 0.662 for the first, second, and third stages, respectively. The correspond-

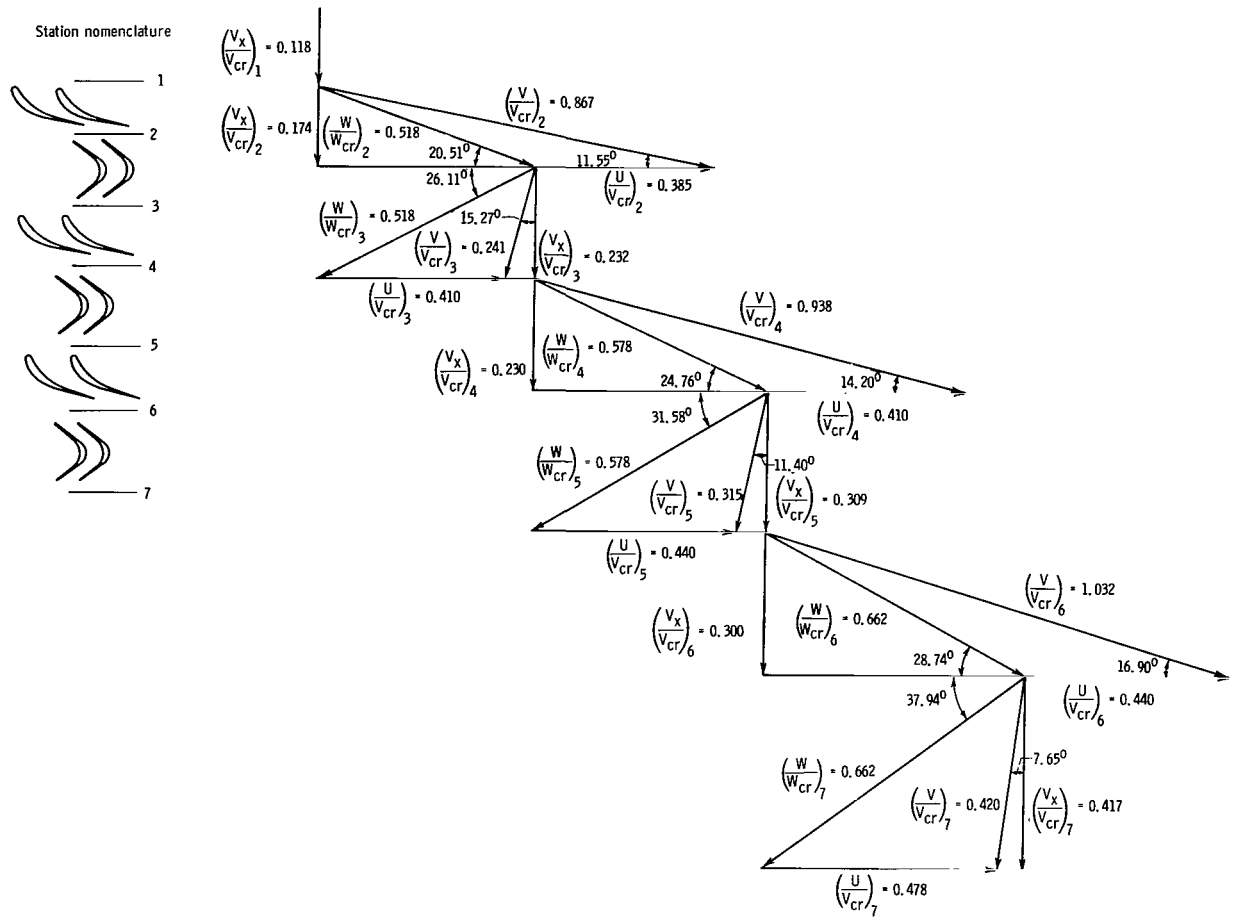


Figure 1. - Free-stream velocity diagrams for three-stage turbine.

ing angles of turning are about  $133^\circ$ ,  $124^\circ$ , and  $113^\circ$ . Further, it is seen that each stage was designed for a small amount of negative exit whirl.

For a constant mean-section diameter of 3.0 inches, the velocity diagrams, as described, require an increase in flow passage height from inlet to exit of the turbine, with blade heights of 0.1, 0.154, and 0.260 inch for the first, second, and third stages, respectively.

Velocity diagrams required for the blade design were computed from the adjacent free-stream velocity diagrams and were based on the following assumed conditions between adjacent stations:

- (1) No change in the tangential component of velocity
- (2) Continuity and no loss in total pressure

The diagrams for the three stators were computed based on 34 blades (solidity of 1.57) with a trailing-edge radius of 0.0025 inch. The calculations for the rotors were based on 75, 85, and 80 blades for the first, second, and third stages, respectively, all rotors having a leading-edge radius of 0.005 inch and a trailing-edge radius of 0.0025 inch.

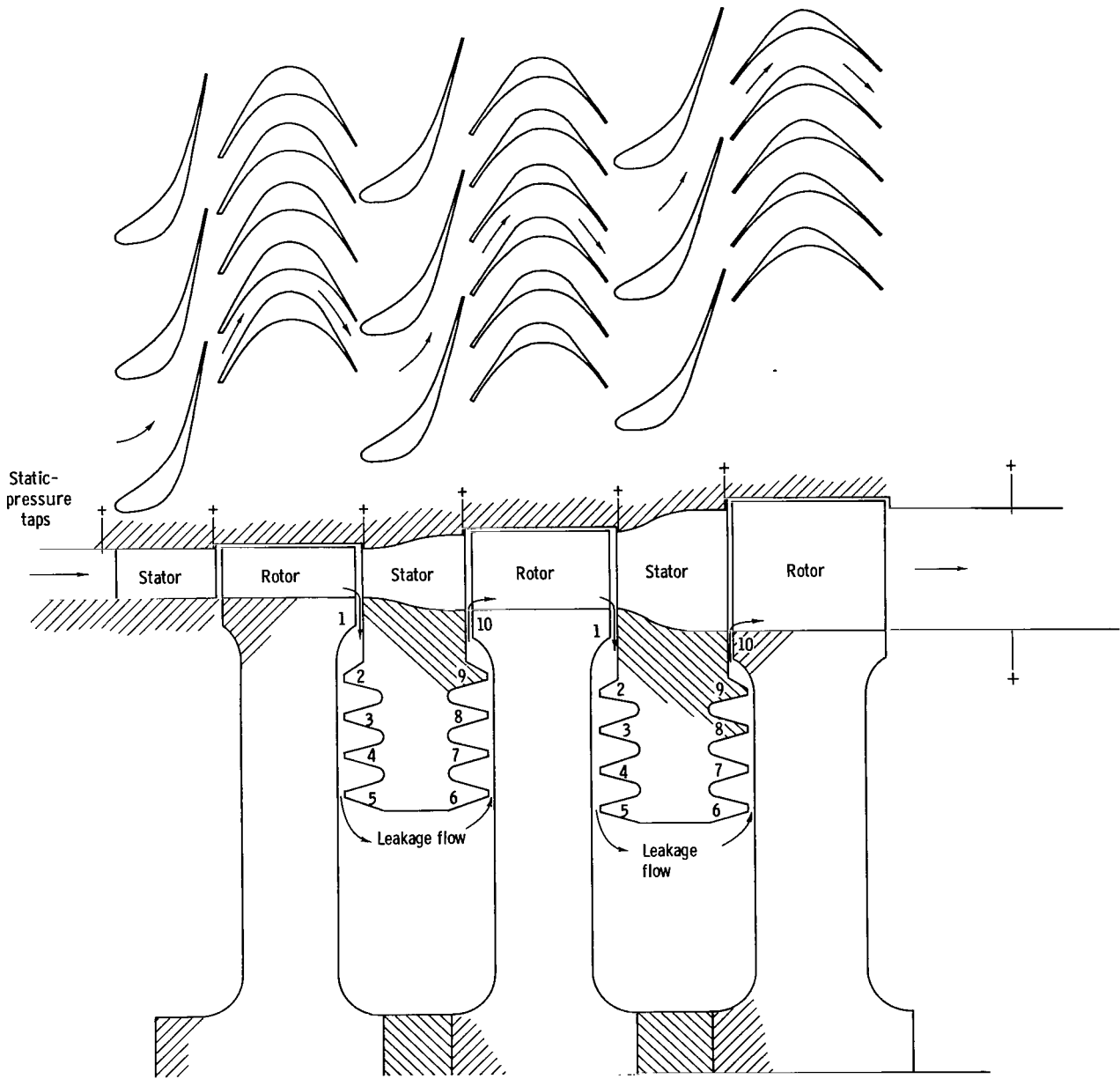
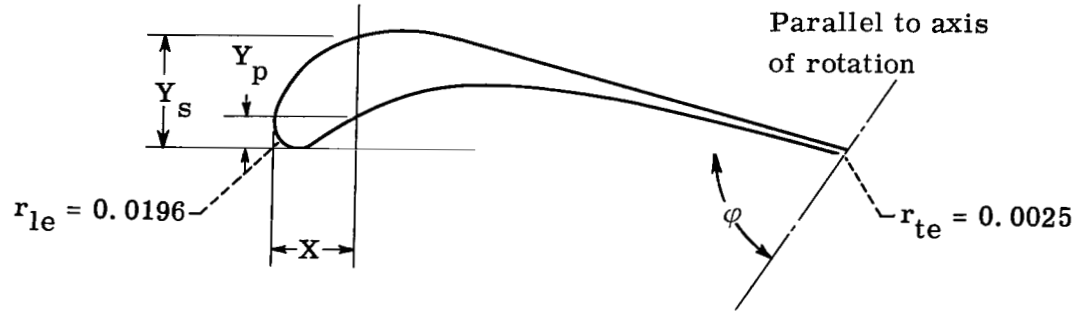


Figure 2. - Blading and seal configuration of three-stage turbine.

## Stators

The design procedure for the stators was generally the same as that for those of reference 1. As stated previously, however, the velocity diagrams for the subject turbine required that the flow passage increase in height from inlet to exit of the turbine. As shown in figure 2, this increase in height in the second- and third-stage stators was

TABLE I. - STATOR-BLADE COORDINATES

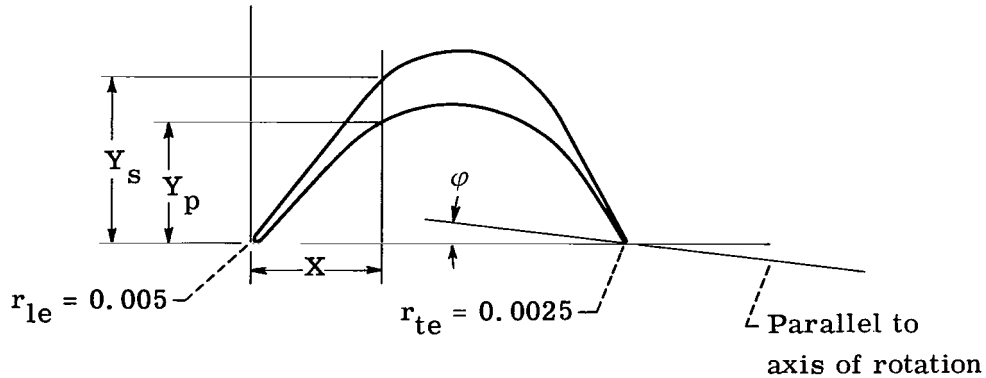


X, in.	$Y_p$ , in.	$Y_s$ , in.
0.000	0.020	0.020
.020	.000	.060
.040	.010	.077
.060	.023	.086
.080	.034	.090
.100	.041	.091
.120	.046	.090
.140	.049	.086
.160	.050	.082
.180	.050	.076
.200	.049	.071
.220	.047	.065
.240	.044	.059
.260	.041	.054
.280	.037	.048
.300	.033	.043
.320	.028	.037
.340	.023	.031
.360	.019	.026
.380	.013	.020
.400	.008	.014
.420	.003	.0088
.435	.0025	.0025

Stage	$\phi$
1	$61^{\circ}45'$
2	$59^{\circ}2'$
3	$56^{\circ}18'$



TABLE II. - ROTOR-BLADE COORDINATES



Stage								
First			Second			Third		
$\varphi$								
5°54'			6°53'			5°32'		
X, in.	Y <sub>p</sub> , in.	Y <sub>s</sub> , in.	X, in.	Y <sub>p</sub> , in.	Y <sub>s</sub> , in.	X, in.	Y <sub>p</sub> , in.	Y <sub>s</sub> , in.
0.000	0.005	0.005	0.000	0.005	0.005	0.000	0.005	0.005
.020	.020	.040	.020	.017	.033	.020	.015	.031
.040	.052	.073	.040	.043	.060	.040	.040	.056
.060	.077	.107	.060	.067	.087	.060	.062	.081
.080	.095	.137	.080	.085	.111	.080	.079	.104
.100	.108	.159	.100	.098	.131	.100	.091	.123
.120	.116	.173	.120	.106	.145	.120	.099	.136
.140	.120	.181	.140	.111	.152	.140	.104	.143
.160	.121	.183	.160	.111	.154	.160	.106	.145
.180	.118	.179	.180	.108	.150	.180	.104	.142
.200	.111	.168	.200	.100	.139	.200	.099	.134
.220	.097	.144	.220	.087	.118	.220	.090	.119
.240	.076	.103	.240	.067	.085	.240	.076	.097
.260	.040	.054	.260	.034	.046	.260	.057	.070
.2810	.0025	.0025	.2816	.0025	.0025	.280	.032	.041
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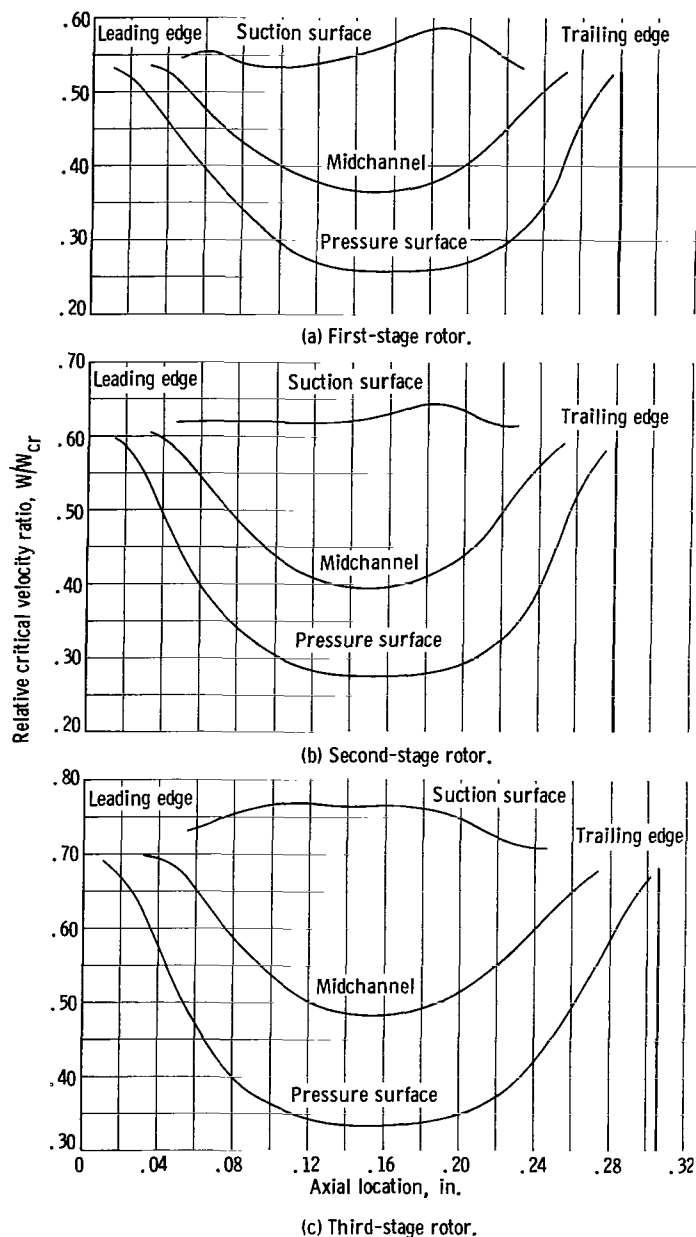


Figure 3. - Design rotor midchannel and surface velocity distributions as functions of axial location.

accomplished by a smooth increase in stator height from the leading edge to a point immediately upstream of the throat. The blade profile as designed for the first-stage stator was also used for the other two stages at slightly different stagger angles. Analysis of the flow channels with a new stagger angle indicated no large changes in surface velocity distribution. The diffusion on the suction surface of each of the three stators was zero. The stator-blade profile and stator-flow passages are shown in figure 2. The blade coordinates are presented in table I.

## Rotors

The turbine rotors were designed with the same procedure as that used in the design of the rotors of reference 1. The rotor-blade profiles and flow passages are shown in figure 2. As seen in the figure, the blades of the three rotors extended into a recess in the turbine casing. This recess allowed for an extension of about 0.003 inch for the first rotor and 0.012 inch for the other two rotors with a tip clearance of about 0.006 inch. As reported in reference 4 some extension of the blades into a recess is effective in minimizing tip clearance losses. Also, the rotor-blade leaving losses are minimized by the thin trailing edges

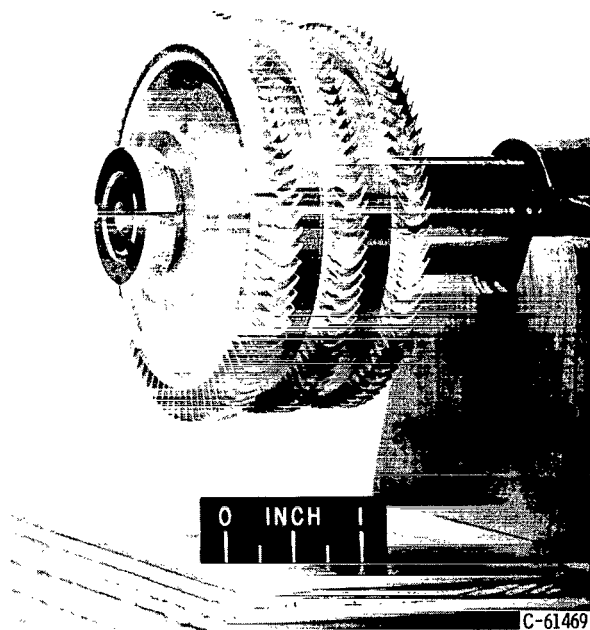


Figure 4. - Turbine rotors.

(0.005 in.). The coordinates for the three rotors are presented in table II. The blade-surface and midchannel velocity distributions are shown in figures 3(a), (b), and (c) for the first, second, and third rotors, respectively. As seen in the figure, all rotors were designed for low values of suction-surface diffusion parameter  $D_s$  with respective values of 0.099, 0.079, and 0.112 for the first, second, and third stages. The corresponding values of pressure-surface diffusion parameter  $D_p$  are 0.520, 0.542, and 0.519, giving values for total diffusion  $D_{tot}$  of 0.619, 0.621, and 0.631. These values of diffusion were obtained by designing for solidities of 2.24, 2.54, and 2.61. The increase in height of

the flow channel between the first and third stages is reflected in an increase in rotor-blade aspect ratio  $A$  from a comparatively low value for the first stage to a near optimum value for the third stage. These values are 0.356, 0.547, and 0.847 for the first, second, and third stages, respectively. A photograph of the three rotor assemblies is shown in figure 4.

## EFFECT OF SEAL LEAKAGE ON PERFORMANCE

In the preceding section covering the aerodynamic design of the subject three-stage turbine, seal leakage was assumed to be less than 1 percent of total turbine flow. Therefore, its effect on turbine performance was neglected in the design because the additional loss due to this leakage would be within the accuracy of the loss coefficients used in predicting the efficiency. In the mechanical design of the turbine, however, seals for the second- and third-stage stator were selected that resulted in seal leakage flows in excess of that assumed in the aerodynamic design of the turbine. Accumulation of fabrication tolerances also resulted in seal clearances that were much larger than design and resulted in still larger seal leakage flows. The amount of seal flows and its effect on turbine performance was calculated as described in appendix B. The seal leakage flows were computed to be 13.3 and 5.3 percent of total turbine flow. It was also computed in appendix B that a 13.3-percent deficit in second-stage weight flow would reduce the work

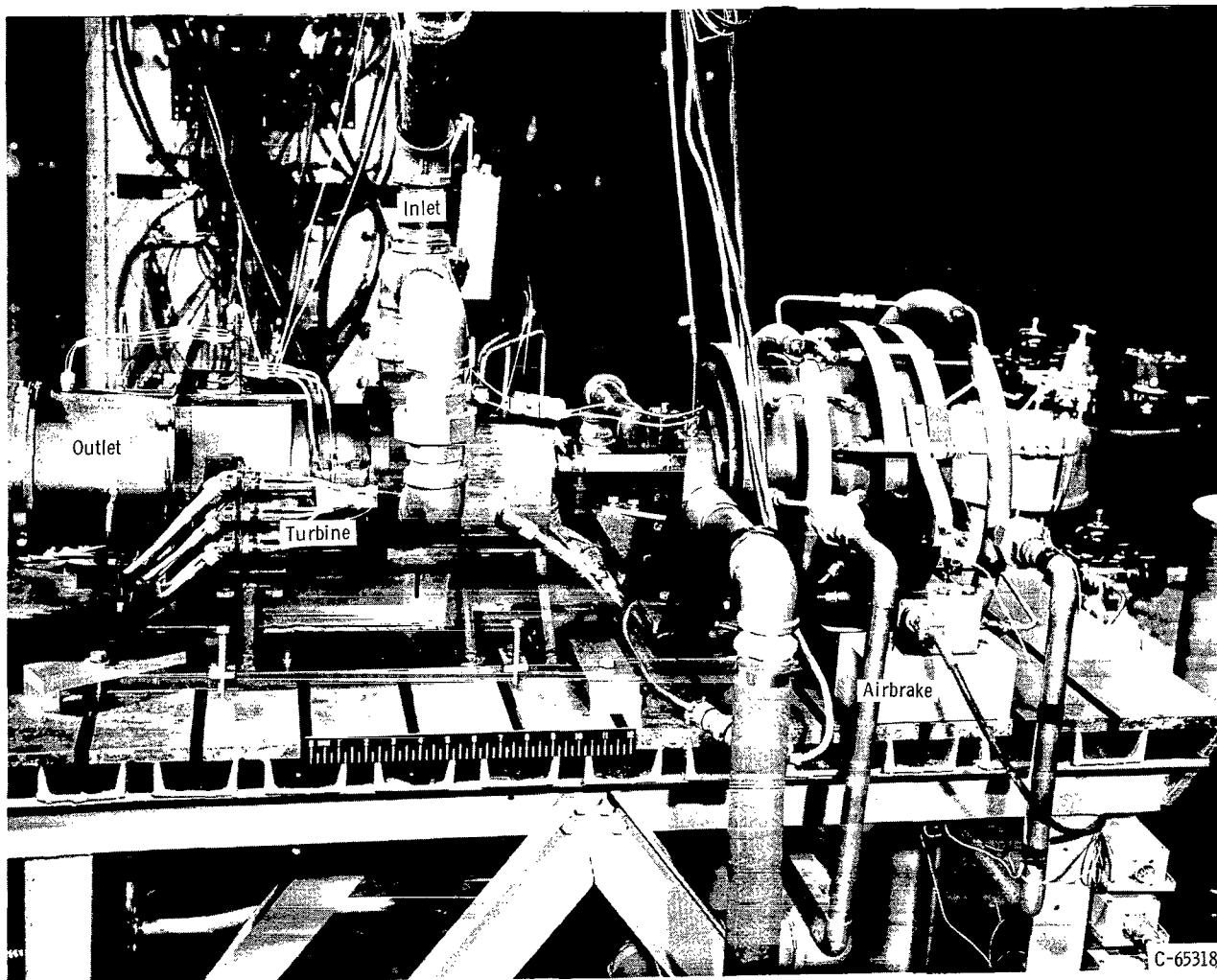


Figure 5. - Turbine test apparatus.

of that stage to 12.60 Btu per pound. Further, a 5.3-percent deficit in third-stage weight flow, including the effect of additional reheat due to inefficiency of the second stage, would reduce the third-stage work output to 14.03 Btu per pound. Thus, the overall turbine work output would be reduced to 41.16 Btu per pound with the associated overall total-to-static efficiency reduced to 0.65. The effect of 1-percent seal leakage through both seals would reduce the overall work output to 43.3 Btu per pound and the static efficiency would still be about 0.69.

## APPARATUS, INSTRUMENTATION, AND PROCEDURE

The apparatus, instrumentation, and procedure used in the investigation of the sub-

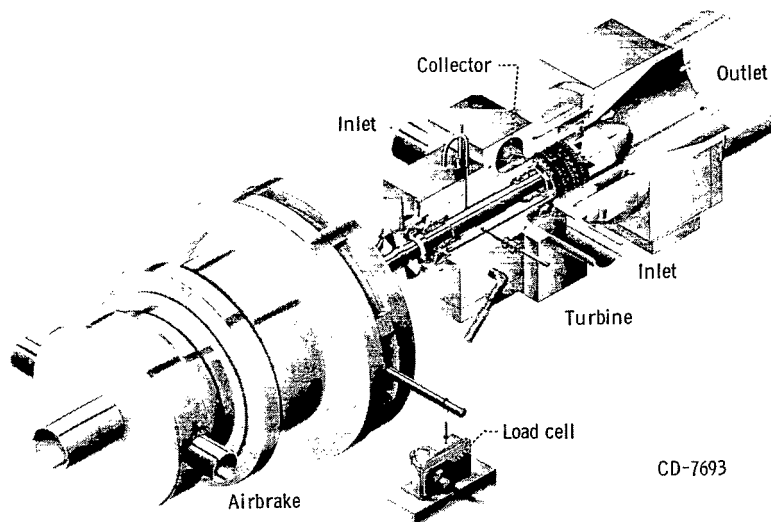


Figure 6. - Cutaway view of turbine coupled to airbrake.

ject turbine are essentially the same as that used in the investigation reported in reference 1. Figure 5 shows a photograph of the experimental turbine installation. The power absorption and measuring device was not the same as that used in the previous investigation but one with the same principle of operation. A description and detailed cutaway view of the air brake is presented in reference 5. A cutaway drawing of the turbine assembly is shown in figure 6. The air enters the turbine through the inlet collector, passes through three stages and into the output pipe from which it is discharged into the laboratory exhaust system.

Instrumentation differed very little from that of reference 1. Since the subject turbine was a full-admission unit, fewer static-pressure taps were required. The turbine-inlet total pressure was measured with two static-pressure taps, which were located in the inlet collector (station 0)  $180^\circ$  apart at points of low Mach number where the total-to-static pressure ratio was assumed equal to 1.0. Turbine overall performance was based on an average of the static pressures measured with eight static-pressure taps located 0.25 inch downstream of the trailing edge of the last-stage rotor blade (fig. 2, p. 5) and equally spaced around the annulus with four on the inner wall and four on the outer wall.

Static pressures at the inlet and exit of each stator-blade row (fig. 2) were also measured with two taps located on the outer wall at points  $180^\circ$  apart, midway between the blades and just at the edge of the rotor recess. These pressures were not considered reliable, however, because of flow effects around the edge and the effects of the rotor moving in the recess.

Experimental data on the three-stage turbine were taken over a range of inlet-total to exit-static pressure ratios and rotative speeds from 9.4 to 15.5 and 13 000 to 31 000

revolutions per minute, respectively. The inlet-total temperature and total pressure were maintained at about  $610^{\circ}$  R and approximately 50 pounds per square inch absolute, respectively. Data above 28 000 revolutions per minute were obtained with the inlet temperature reduced to approximately  $543^{\circ}$  R in order to keep rotative speed below 31 000 revolutions per minute and still achieve design equivalent rotative speed. This was done because of a scatter in the data above 31 000 revolutions per minute, which was attributed to variations in bearing and seal friction.

Experimental data on the first stage of the three-stage turbine operated as a single-stage turbine were obtained at an inlet-total temperature and pressure of  $545^{\circ}$  R and 108 pounds per square inch absolute, respectively. The rotative speed and total-to-static pressure ratio were varied from 12 000 to 33 500 revolutions per minute and 1.75 to 2.32, respectively.

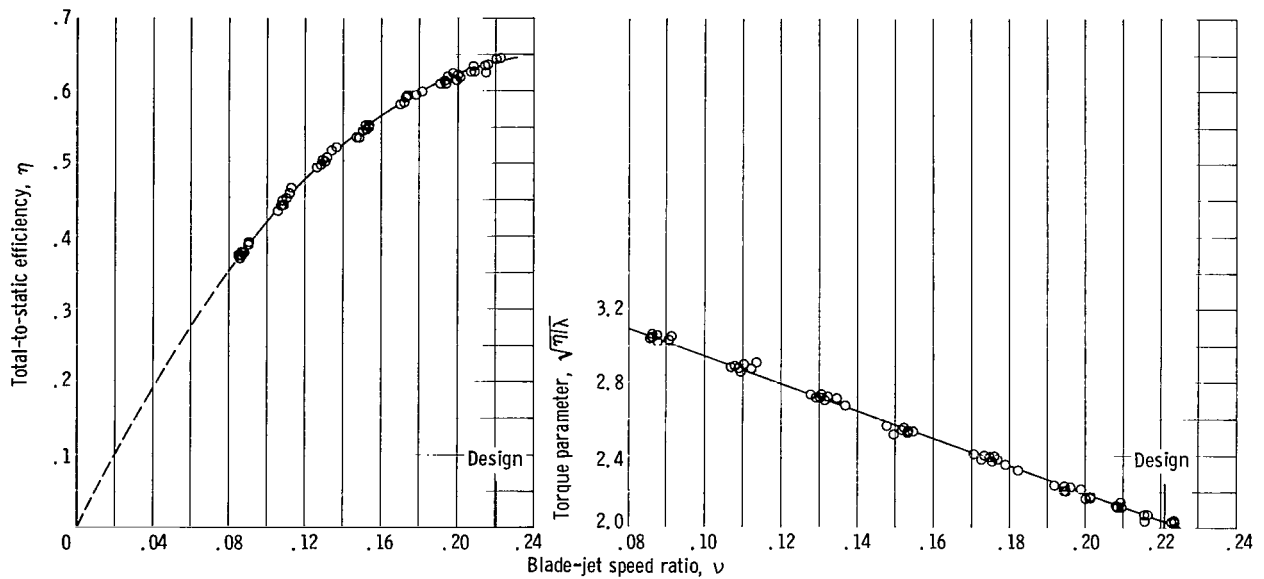
## RESULTS AND DISCUSSION

The results of the experimental investigation of the 3.0-inch-mean-diameter three-stage turbine and the first stage as a single-stage turbine are presented. The results of the first-stage investigation are presented to verify the level of loss coefficient used in the design of turbines in this size class. A comparison of overall performance is made with that of the two-stage partial-admission reference turbine designed for the same rotative speed, flow, and power requirements.

### Three-Stage Overall Performance

The overall performance characteristics of the subject three-stage turbine are presented in figure 7, where total-to-static efficiency and torque parameter are plotted against blade-jet speed ratio. Figure 7(a) shows that, at the design blade-jet speed ratio of 0.221, an efficiency of about 0.64 was obtained, which compares favorably with the value of 0.65 computed for this turbine with second- and third-stage stator seal leakages of 13.3 and 5.3 percent, respectively. Figure 7(b) shows that the torque-speed curve over the range investigated is conventional in that it is linear.

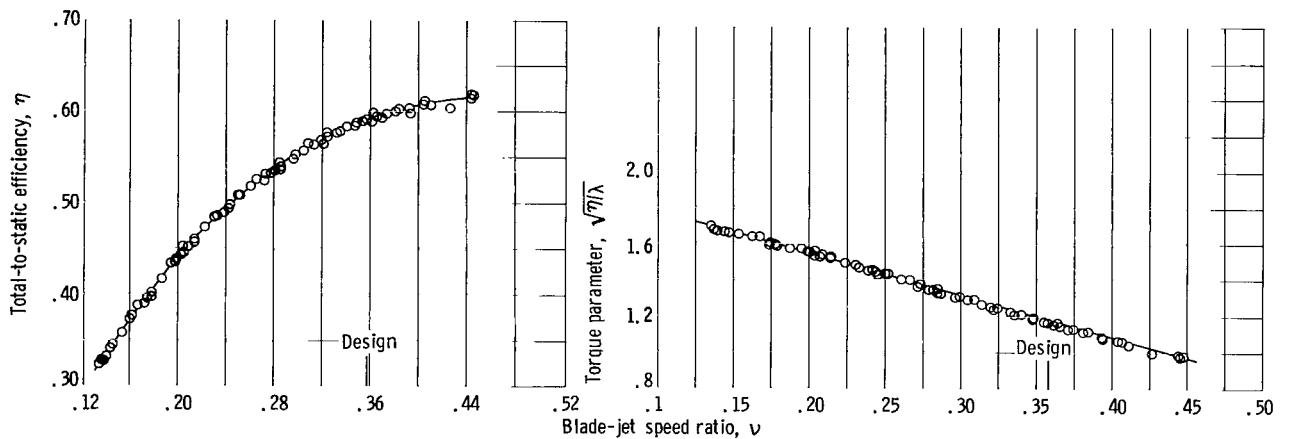
The equivalent weight flow  $\epsilon w \sqrt{\theta}/\delta$  passed by the turbine was 0.0614 pound per second at design speed and design pressure ratio or about 2.3 percent greater than design weight flow. A calculation of stator-exit velocity diagrams to match continuity, first-stage work output, measured first stage rotor-exit static pressure, and rotor- and stator-exit blade angles, indicated that the flow coefficient was about 0.97, whereas if the stator is assumed to be choked, the computed flow coefficient was 0.95. These values



(a) Variation of total-to-static efficiency with blade-jet speed ratio.

(b) Variation of torque parameter with blade-jet speed ratio.

Figure 7. - Performance characteristics of three-stage turbine.



(a) Variation of total-to-static efficiency with blade-jet speed ratio.

(b) Variation of torque parameter with blade-jet speed ratio.

Figure 8. - Performance characteristics of first stage.

bracket the flow coefficient of approximately 0.96, which appears reasonable for stators in this size class.

## First-Stage Overall Performance

The overall performance characteristics of the first stage operating as a single-stage turbine are presented in figure 8 where total-to-static efficiency and torque param-

eter are plotted against blade-jet speed ratio. Figure 8(a) shows that at design blade-jet speed ratio of 0.357, the total-to-static efficiency is 0.59 compared with 0.60 used in the design. The torque-speed characteristics plotted in figure 8(b) indicate a linear characteristic with an extrapolated zero speed value approximately 1.5 times the design value. The close agreement between the experimental and design efficiency of the first stage verified the value of loss coefficient used in the design of the three-stage turbine. These results further indicated that if the seal leakages were substantially reduced, the three-stage turbine should perform near an efficiency of 0.69.

### Comparison of Overall Performance of Three-Stage Turbine with That of Two-Stage Partial-Admission Turbine

Table III is included to facilitate the comparison of the subject turbine with the reference turbine. The configuration of the reference two-stage turbine was varied by changing the distance between the trailing edge of the first-stage rotor and the leading edge of the second-stage stator. Figure 9 presents a diagram of the blading of the reference turbine showing the two extreme configurations tested. In reference 1 the total-to-static efficiency at design blade-jet speed ratio was plotted as a function of turbine length. Tur-

TABLE III. - TURBINE PERFORMANCE COMPARISON

Stage	Subject turbine	Reference turbine
	Number of stages	
	3	2
	Mean diameter, in.	
	3.0	3.75
First Second Third	Blade height, in.	
	0.10	0.217
	.154	.217
	.260	---
Admission, deg		
First Second Third	360 360 360	112.8 360 ---

bine length was measured from first-stage-stator leading edge to last-stage-rotor trailing edge and was plotted as the significant parameter because of its effect not only on efficiency but also on overhung weight and therefore critical speed. Figure 10 presents a replot of the figure from reference 1 that indicates a large improvement in efficiency as turbine length is increased to about 2 inches. Increases in performance with increase in length above 2 inches is also indicated but at a much lower rate. This increase in efficiency with length was attributed to reduced circumferential gradients into the second stage. An efficiency of 0.64 for the subject turbine with high seal leakage flows is plotted in figure 10 with its associated length. Comparison of the performance of the subject turbine with the reference turbine indicates that the efficiency of the subject three-stage turbine is three points higher than that of the reference turbine with the same length. The predicted efficiency for the subject three-stage turbine with substantially reduced seal leakage flow is also shown in figure 10.



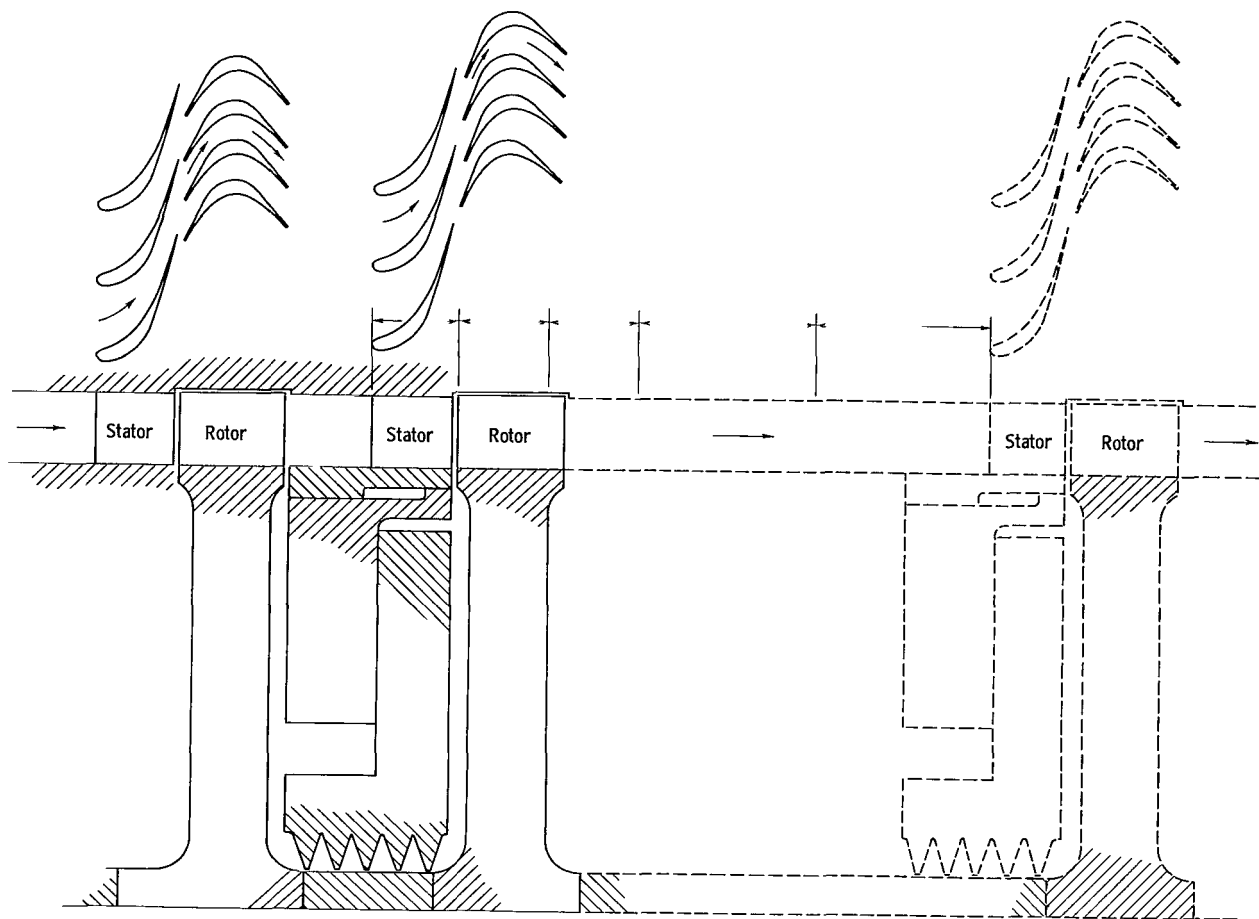


Figure 9. - Two-stage reference turbine configuration showing extreme and intermediate positions of second stage.

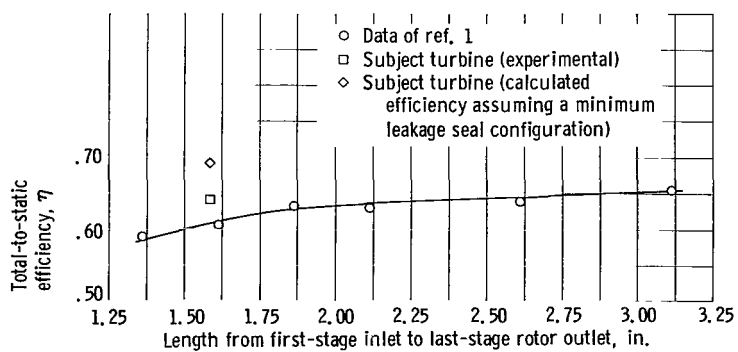


Figure 10. - Comparison of design-point static efficiency of subject turbine with that of two-stage partial admission turbine of reference 1.

From the figure it can be concluded that, in general, the subject three-stage turbine with its high-seal leakage performed as well as the best configuration of the two-stage turbine. Further, it is expected that the three-stage turbine with substantially reduced seal leakage flow would be three to four points higher in efficiency than the two-stage turbine.

It should be noted that this comparison should not be generalized beyond the scope of this investigation because changes in the arc of admission of the first and second stage of the two-stage turbine could have considerable effects on these results.

## SUMMARY OF RESULTS

The performance is presented for a 3.0-inch-mean-diameter full-admission three-stage turbine and for the first stage as a single-stage turbine. A comparison is made between the three-stage turbine and a two-stage partial-admission turbine, which had the same design requirements of rotative speed, specific work output, and weight flow. The pertinent results of the investigation are:

1. At the design blade-jet speed ratio of 0.221, a total-to-static efficiency of 0.64 was obtained with a high-leakage-seal configuration. This value compares favorably with an efficiency of 0.65, which was calculated from an aerodynamic efficiency of 0.69 to account for seal leakages of 13.3 and 5.3 percent of turbine flow in the second- and third-stage stator seals, respectively.

2. At the design blade-jet speed ratio of 0.357, a total-to-static efficiency of 0.59 was obtained for the first stage operated as a single-stage turbine compared with 0.60 used in the design. The close agreement between design and experimental results verified the level of loss coefficient used in the design procedure, and also indicated that, if the seal leakage of the three-stage turbine was substantially reduced, an efficiency near 0.69 would be obtained.

3. Comparison of the performance of the three-stage turbine with that of the reference two-stage turbine indicates that the performance of the three-stage turbine with the high leakage seals performed as well as the best of the various configurations of the two-stage turbine. Further, if the seal leakages were substantially reduced, it is expected that the three-stage turbine would be three to four points higher in efficiency than the best two-stage configuration.

Lewis Research Center,  
National Aeronautics and Space Administration,  
Cleveland, Ohio, December 3, 1964.

## APPENDIX A

### SYMBOLS

$\mathcal{A}$	blade aspect ratio, $b/c$
$b$	blade height, ft
$C_b$	loss coefficient determined from experimental data in ref. 3, $C_b = 0.0216$
$c$	blade chord, ft
$D_p$	pressure-surface diffusion parameter, (blade inlet relative velocity) - (minimum blade surface relative velocity) blade inlet relative velocity
$D_s$	suction-surface diffusion parameter, (maximum blade surface relative velocity) - (blade outlet relative velocity) maximum blade surface relative velocity
$D_{tot}$	sum of pressure- and suction-surface diffusion parameters, $D_p + D_s$
$g$	gravitational constant, $32.17 \text{ ft/sec}^2$
$\Delta h$	specific work output, Btu/lb
$hp$	horsepower
$J$	mechanical equivalent of heat, $778 \text{ ft-lb/Btu}$
$p$	absolute pressure, lb/sq ft
$U$	blade velocity, ft/sec
$V$	absolute gas velocity, ft/sec
$W$	relative gas velocity, ft/sec
$w$	weight flow rate, lb/sec
$\gamma$	ratio of specific heats
$\delta$	ratio of turbine-inlet total pressure to that of U. S. standard sea-level atmosphere, $p'_0/2116.22$
$\epsilon$	function of $\gamma$ used in relating weight flow to inlet conditions at U. S. standard sea-level atmosphere, $\frac{0.740}{\gamma} \left( \frac{\gamma + 1}{2} \right)^{\gamma/(\gamma-1)}$
$\eta$	adiabatic efficiency, ratio of blade power to ideal blade power based on inlet-total to exit-static pressure ratio

- $\theta_{cr}$  squared ratio of critical velocity at turbine inlet to critical velocity at U. S. standard sea-level atmosphere,  $(V_{cr,0}/1019.46)^2$
- $\lambda$  speed-work parameter,  $U_m^2/(gJ \Delta h)$
- $\nu$  blade-jet speed ratio,  $U_m/\sqrt{2gJ \Delta h_{id}}$

**Subscripts:**

- cr conditions at Mach number of unity
- id ideal
- m mean radius
- x axial direction
- 0 station at turbine inlet collector (fig. 6, p. 11)
- 1 station at turbine inlet (fig. 1, p. 4)
- 2 station between first-stage stator and first-stage rotor
- 3 station between first-stage rotor and second-stage stator
- 4 station between second-stage stator and second-stage rotor
- 5 station between second-stage rotor and third-stage stator
- 6 station between third-stage stator and third-stage rotor
- 7 station downstream of third-stage rotor

**Superscript:**

- ' absolute total state

## APPENDIX B

### METHOD OF DETERMINING EFFECT OF SEAL LEAKAGE ON PERFORMANCE

In the section covering the aerodynamic design of the subject turbine, seal leakage was assumed to be less than 1 percent of turbine flow; therefore, its effect on turbine performance was neglected because the additional loss due to leakage would be within the accuracy of the loss coefficients used in predicted design aerodynamic efficiency.

In the mechanical design, the turbine seals for the second- and third-stage stator were selected that resulted in seal leakage flows in excess of that assumed in the aerodynamic design of the turbine. Accumulation of fabrication tolerances also resulted in seal clearances that were much larger than design. This resulted in seal leakage flows that were greatly in excess of that assumed. The following method of analysis was aimed at determining the amount and effect of the larger leakage flows on the predicted performance of the subject three-stage turbine. The analysis will consist of computing the seal leakage from available coefficients and then calculating the effect of the leakage on turbine performance.

An accurate calculation of a labyrinth seal leakage flow depends on empirical coefficients. No data could be found that matched the complete seal configuration used. Included in figure 2 (p. 5) is a sketch of the seal configuration used. Therefore, discharge coefficients for single radial glands as presented in reference 6 and kinetic energy carry-over coefficients from reference 7 were used. It should be noted that the discharge coefficients for the single radial gland were determined for the nonrotating case. The kinetic energy carry-over coefficient from gland 2 to 3, 3 to 4, 4 to 5, 6 to 7, 7 to 8, and 8 to 9 (fig. 2) was determined for flow through annular orifices in series from boundary layer considerations (ref. 7). The effect of flow being centrifuged on the surface of the rotor disk, against the direction of leakage flow on one side of the seal and with the direction of leakage flow on the other side, on the coefficients does not appear to be easily analyzed. It is probable that the rotation of the rotor could cause such high turbulence between the glands of this seal configuration that the kinetic energy carry-over may be much less than that predicted from reference 7. It is expected that the seal leakage computed with these coefficients represents a maximum flow.

The weight flow was computed by an iterative process that involved the coefficients and a computation from gland to gland to match the design static pressures at the hub of the stator inlet and exit. The computation assumed no heat addition, and the kinetic energy between glands 1 and 2, 5 and 6, and 9 and 10 is completely dissipated. The discharge coefficient and kinetic energy carry-over coefficients for the other glands were determined from the geometry and the references. The resultant seal leakage flows for

the second- and third-stage seals were computed to be 13.3 and 5.3 percent, respectively, of total turbine flow.

The effect of seal leakage flows on turbine performance was determined as follows. It was assumed that the flow passing through the stator expanded to the design velocity diagrams. This assumption appears to be valid since the leakage flow reentering between the stator and rotor must cause the stator flow to separate at the hub thus reducing the effective throat area of the stator. It is evident that there is a deficit in tangential momentum due to the deficit in weight flow. The seal leakage flows reentering the main stream between the stator and rotor is considered to have zero tangential momentum. When the two streams are mixed, the final tangential momentum is considered to be a mass average of the two streams. It is recognized that the leakage flows will have some tangential momentum due to viscous effect on the rotor; however, the momentum imparted to the leakage flow by the rotor actually cancels an equal value of momentum imparted to the rotor by the gas. Therefore, the assumption of zero tangential momentum for the seal leakage flow should be valid.

The effect of 13.3 percent second-stage seal leakage was computed to decrease the second-stage work to 12.60 Btu per pound. Including the effect of the additional reheat, the third-stage work reduced to 14.03 Btu per pound. This resulted in an overall turbine work output of 41.16 Btu per pound and a resultant computed static efficiency of 0.65. The effect of 1-percent seal leakage in both seals would reduce the overall work to 43.3 Btu per pound and the corresponding efficiency would be about 0.69.

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